*Practical class 8*

*Topic:* ***Check of ACS on stability by Hurwitz, Mikhailov and Nyquist criteria***

I. *Check of Dynamic System on stability by criterion Hurwitz*

The algebraic criterion of stability Hurwitz applies to the closed systems. However we know transfer function *W (s)* of open-ended (open-look) ACS.

*The integrated Algorithm*

1. It is necessary "”mentally" to close system by unit negative feedback, to find transfer function of the closed system under the formula:

$$Wcl\ (s) = \frac{W_{open}(s)}{1+W_{open}(s)}.$$

2. Write down the characteristic equation of the closed system:

$$a_0 s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n = 0.$$

3. Make the determinant Hurwitz of factors of the characteristic equation as follows:

$$\Delta(n*n) = \begin{bmatrix} a_1 & a_3 & a_5 & \ldots & 0 \\ a_0 & a_2 & a_4 & \ldots & 0 \\ 0 & a_1 & a_3 & \ldots & 0 \\ 0 & a_0 & a_2 & \ldots & 0 \\ .. & .. & .. & .. & \ldots \\ 0 & 0 & 0 & \ldots & a_n \end{bmatrix} > 0,$$ where ***n***-order of system.

4. From the formulation of criterion of stability Hurwitz follows, that at $a_0 > 0$ for stability of system it is necessary and enough, that $\Delta_1 = a_1 > 0$,

$$\Delta_2 = \begin{bmatrix} a_1 & a_3 \\ a_0 & a_2 \end{bmatrix} > 0, \quad \Delta_3 = \begin{bmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{bmatrix} > 0 \quad \text{etc.}$$

5. We carry out the analysis and we find conditions which it guarantees stability of closed system according to criterion Hurwitz.

*Example* Check up dynamic system on stability by criterion Hurwitz if the transfer function of open-ended ACS is given of the following kind:

$$W\ (s) = \frac{K(T_2 s+1)}{T_3 s(T_1 s+1)^2},$$ where $K>0$, $T_I>0$ $\forall$ $i=\overline{1,3}$.

*Algorithm and solution*

1. The transfer function of system closed unit negative feedback is determined:

$$Wcl\ (s) = \frac{K(T_2 s+1)}{T_3 s(T_1 s+1)^2 + K(T_2 s+1)}.$$

2. The characteristic equation of the closed system enters the name:

$$Q_2(s) = T_3 s\ (T_1 s+1)^2 + K(T_2 s+1) = 0$$

$$T_1^2\ T_3 s^3 + 2T_1\ T_3 s^2 + (T + KT_2)s + K = 0.$$

3) Make the determinant Hurwitz of factors of the characteristic equation as follows:

$$a_0 = T_1^2 T_3; \quad a_1 = T_1 T_3; \quad a_2 = T_3 + KT_2; \quad a_3 = K;$$

$$\Delta(3*3) = \begin{bmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{bmatrix} > 0 \qquad \Delta = \begin{bmatrix} 2T_1T_3 & K & 0 \\ T_1^2 T_3 & (T_3 + KT_2) & 0 \\ 0 & 2T_1T_3 & K \end{bmatrix} > 0.$$

4. The condition of stability is checked according to the formulation of criterion Hurwitz n:

$a_0 = T_1^2 T_3 > 0$, as on a condition is given $Ti > 0 \quad \forall\ i = \overline{1.,3}$ ;

$\Delta_1 = a_1 = T_1 T_3 > 0$, as on a condition is given $T\text{i} > 0 \quad \forall\ i = \overline{1.,3}$ ;

$$\Delta_2 = \begin{bmatrix} 2T_1T_3 & K \\ T_1^2 T_3 & (T_3 + KT_2) \end{bmatrix} > 0$$

$$2T_1\ T_3(T_3 + KT_2) - KT_1^2 T_3 > 0$$

$K < \dfrac{2T_3}{T_1 - 2T_2}$ , (*) and $T_1 > 2T_2$ (**), as $K > 0$.

*The answer:* this closed system is steady on criterion Hurwitz at performance of the conditions (*) and (**).

*The note:* $\Delta_3$ it is possible to not calculate, as soon as is found (*) and (**) the parity between factor of amplification and constant time, i.e. $K$ and $T_i$ ($\forall$ i=$\overline{1,n}$), at which the system is steady on criterion Hurwitz.

II. *Check of Dynamic System on stability by criterion Mikhailov*

The frequency criterion of stability Mikhailov applies to the closed systems. However we know transfer function *W (s)* of open-ended (open-look) ACS.

Definition: *For stable closed-loop system of any order it is necessary and sufficient, that hodograph curve of vector* $D(j\omega)$, *which defines Mikhailov curve, meets the following requirements:*

a) starts at positive real axis;

b) sequentially passes *“n”* quadrants;

c) have an angle of rotation $\varphi = n\frac{\pi}{2}$ (frequency changes from 0 to infinity);

d) goes to infinity in *n*th quadrant where *“n”* is the order of characteristic equation.

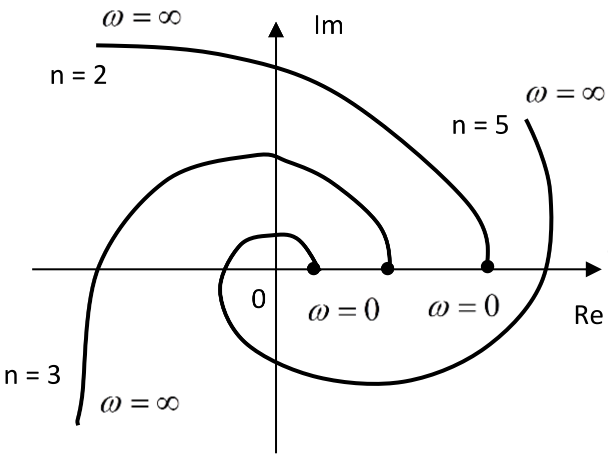



Fig. 1 – Hodograph curve of a stable ACS

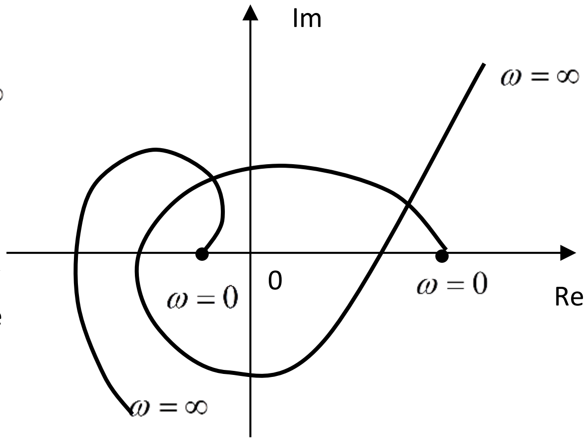



Fig. 2 – Hodograph curve of an unstable ACS

*The integrated Algorithm*

1. It is necessary "”mentally" to close system by unit negative feedback, to find transfer function of the closed system under the formula:

$$Wcl\ (s) = \frac{W_{open}(s)}{1+W_{open}(s)}.$$

2. Write down the characteristic equation of the closed system:

$$a_0 s^n + a_1 s^{n-1} + \ldots + a_{n-1} s + a_n = 0.$$

3. Break the analytical equation of a curve of Mikhailov on real and imaginary parts:

$$D\ (j\omega) = Re\ D\ (j\omega) + Im\ D\ (j\omega).$$

4. Construct a curve of Mikhailov on control points; make conclusions about stability or instability of the given system.

*Example.* Check up dynamic system on stability by frequency criterion Mikhailov, if the transfer function opened ACS is given of the following kind:

$$W(s) = \frac{K(T_2 s+1)}{T_3 s(T_1 s+1)^2}$$ , where $K>0$, $T_I>0 \quad \forall\, i=\overline{1,3}.$

*Algorithm and solution*

1. The transfer function of system closed unit negative feedback is determined:

$$Wcl\ (s) = \frac{K(T_2 s+1)}{T_3 s(T_1 s+1)^2 + K(T_2 s+1)}.$$

2. The analytical equation of a curve of Mikhailov write following:

$Q_2(s) = T_3 s\ (T_1 s+1)^2 + K(T_2 s+1) = 0$ - characteristic equation;

$$D(j\omega) = T_1^2\ T_3 s^3 + 2T_1\ T_3 s^2 + (T_3+KT_2)s + K = /\ s=j\omega/$$

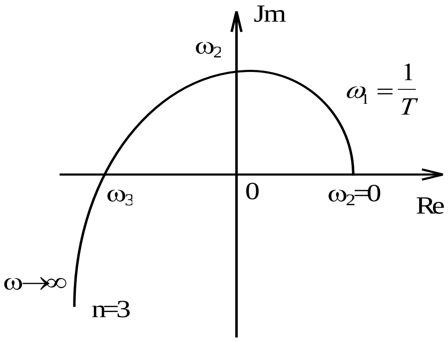
$$= -jT_1^2\ T_3\omega^3 - 2T_1T_3\omega^2 + j(T_3+KT_2)\omega + K = K - 2T_1T_3\omega^2 + j[(T_3+KT_2)\omega -$$
$$-\ \ T_1^2\ T_3\omega^3].$$

3. The analytical equation of a curve Mikhailov's is broken into real and imaginary parts:

$Re\ D(j\omega) = K - 2T_1T_3\omega^2$;

$Im\ D(j\omega) = (T_3+KT_2)\omega - T_1^2\ T_3\omega^3$.

| $\omega$ | $Re\ D\ (j\omega)$ | $Im\ D\ (j\omega)$ |
|---|---|---|
| 0 | $K$ | $0$ |
| $\frac{1}{T_i}$ | $K - 2$ | $K$ |
| $\infty$ | $-\infty$ | $-\infty$ |




*Conclusion*: the system is steady, as hodograph of Mikhailov begins on a positive real axis (coordinate of a point *(K; j0), K>0* on a condition), consistently passes space $\omega_0<\omega_1<\omega_2<\omega_3$ and leaves in infinity in III a space, that coincides with the order of system *n* =3.

III. *Check of Dynamic System on stability by criterion Nyquist*

Criterion itself: *if the point with coordinates* $(-1, j0)$ *in complex plane is not contained within an interior of the GPhFC curve of an open-loop stable dynamic system, then the corresponding closed-loop system is stable* (fig. 3 - 4).

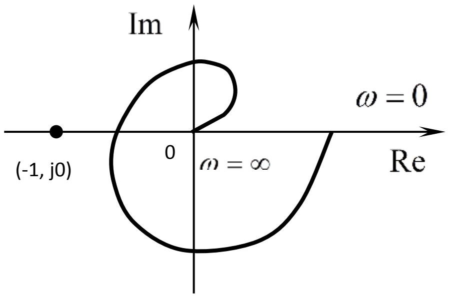



Fig. 3 – GPhFC of an open system

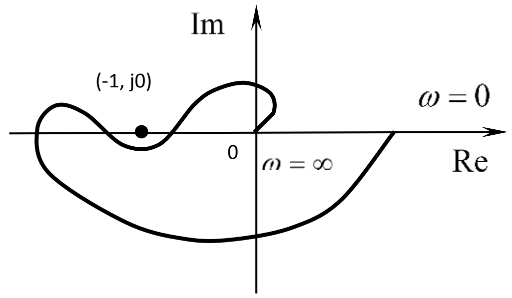



Fig. 4 – Hodograph of a stable system

*This is a necessary and sufficient condition for stability of dynamic system with unit negative feedback.*

*Example.* Check up dynamic system on stability by frequency criterion *Nyquist,* if the transfer function opened ACS is given of the following kind:

$$W(s) = \frac{K(T_2 s + 1)}{T_3 s (T_1 s + 1)^2}, \text{ where } K>0,\ T_I>0\quad \forall i = \overline{1,3}.$$

*Algorithm and solution*

1. We substitute in transfer function of the opened system $s=j\omega$: $W(s)|_{s = j\omega}$

$$\frac{K(T_2 s+1)}{T_3 s(T_1 s+1)^2}\Big|_{s = j\omega} = W(j\omega) = \frac{K(1+jT_2\omega)}{jT_3\omega(1+jT_1\omega)^2}.$$

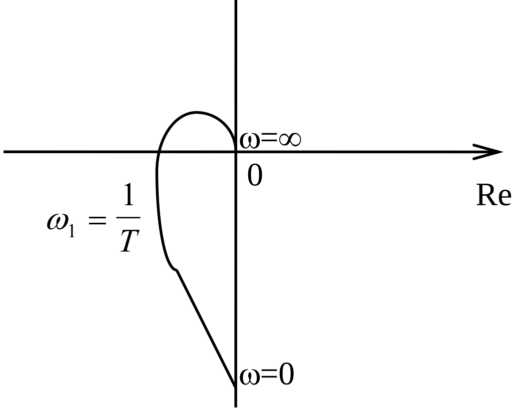
2. We are exempted from ostensibility in a denominator; we break $W(j\omega)$ into real and imaginary parts.

$$W(j\omega) = \frac{jK(1+jT_2\omega)(1-jT_1\omega)^2}{-T_3\omega(1+jT_1\omega)^2(1-jT_1\omega)^2} = \frac{KT_1^2T_2\omega^3+K(2T_1-T_2)\omega+jK[1+T_1(2T_2-T_1)\omega^2]}{-T_3\omega(1+T_1^2\omega^2)^2} =$$

$$= \frac{K[T_1^2T_2\omega^{33}+(2T_1-T_2)\omega]}{-T_3\omega(1+T_1^2\omega^2)^2} + j\frac{K[1+T_1(2T_2-T_1)\omega^2]}{-T_3\omega(1+T_1^2\omega^2)^2}$$

4. We obtain curve $W(j\omega)$ on control points:

| $\omega$ | *Re W(jω)* | *Im W(jω)* |
|---|---|---|
| 0 | 0 | $-\infty$ |
| $\frac{1}{T_i}$ | $-\frac{K}{2}$ | $-\frac{K}{2}$ |
| $\infty$ | 0 | 0 |




*Conclusion:* the closed system is steady as AFH of the opened system does not cover a point with coordinates *(-1; j0),* if $K$<2 (more precisely, if $K < \frac{2T_3}{T_1 - 2T_2}$ and $T_1 > 2T_2$ by Hurwitz's criterion ).

***Task*** Check up dynamic system on stability by Hurwitz, Mikhailov and Nyquist criteria if the transfer function of open-ended ACS is given of the following kind (on variant):

***Variants***

1)

$$W(s) = \frac{k(T_3 s + 1)}{T_2 s(T_1^2 s^2 + 1)},$$

where $k > 0,\ T_i > 0,\ \forall i = \overline{1,3}.$

2)

$$W(s) = \frac{k(T_1 s + 1)}{T_4 s(T_2 s + 1)(T_3 s + 1)},$$

where $k > 0,\ T_i > 0,\ \forall i = \overline{1,3}.$

3)

$$W(s) = \frac{k(T_4 s + 1)}{T_2^2 s^2 (T_3 s + 1)(T_1 s + 1)},$$

where $k > 0,\ T_i > 0,\ \forall i = \overline{1,4}.$

4)

$$W(s) = \frac{k(T_1 s + 1)}{T_2 s(T_3 s + 1)^2},$$

where $k > 0,\ T_i > 0,\ \forall i = \overline{1,3}.$

5)

$$W(s) = \frac{k(T_3 s + 1)}{T_4^2 s^2 (T_2 s + 1)(T_1 s + 1)},$$

where $k > 0,\ T_i > 0,\ \forall i = \overline{1,4}.$

6)

$$W(s) = \frac{k(T_1 s + 1)}{(T_3 s + 1)^2 (T_2 s + 1)^2},$$

where $k > 0,\ T_i > 0,\ \forall i = \overline{1,3}.$

7)

$$W(s) = \frac{k(1+T_1 s)}{(1+T_2 s)(1+T_3 s)^2},$$

where $k > 0,\ T_i > 0,\ \forall i = \overline{1,3}.$

8)

$$W(s) = \frac{k}{T_2^2 s^2 (T_1 s + 1)^2},$$

where $k > 0,\ T_i > 0,\ \forall i = \overline{1,2}.$

9)

$$W(s) = \frac{k(T_2 s + 1)}{(T_3 s + 1)(T_1 s + 1)^2},$$

where $k > 0,\ T_i > 0,\ \forall i = \overline{1,3}.$

10)

$$W(s) = \frac{k(T_1 s + 1)}{T_3 s (T_2 s + 1)^2},$$

where $k > 0,\ T_i > 0,\ \forall i = \overline{1,3}.$

11)

$$W(s) = \frac{k(T_4 s + 1)}{T_2 s (T_3 s + 1)(T_1 s + 1)},$$

where $k > 0,\ T_i > 0,\ \forall i = \overline{1,4}.$

12)

$$W(s) = \frac{k(1+T_2 s)}{s(T_3 s + 1)^2 (T_1 s + 1)},$$

where $k > 0,\ T_i > 0,\ \forall i = \overline{1,3}.$